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Local Voting Protocol for the Adaptation of Airplane's "Feathers" in a Turbulence Flow

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Abstract— **The traditional detailed mathematical description of the motion of complicated complex systems with many transducers/sensors and actuators often leads to a very hard problems involving an extremely high-dimensional state space. However, multi-agent technology can effectively solve many of such problems by replacing the general model of interactions in a complex system with a set of local models and their aggregation (clustering). In this paper we apply the results of the synchronization and consensus achievement within the network control to the flight control tasks when a vast array of sensors and actuators ("feathers") is distributed over the surface of an airplane. The usage possibilities of Local Voting Protocol for the adaptation of airplane's "feathers" in a turbulence flow are examined.**

I. INTRODUCTION

Miniaturization and increased performance of calculators, sensors and actuators can significantly extend the practical applicability of the modern scientific control, identification and estimation theories. At the same time, there are possibilities of intelligent control of complex mechatronic systems during the transition process and turbulence. For example, the clustering of disturbing factors in the environment may lead to changes in the structure of the state space (dimension). For multi-agent systems, on the one hand, when a consensus in the behavior of certain groups of agents is reached the total dimension of the whole state space is reduced. On the other hand, the effect of disturbances can result in misalignment of agents' behavior in a group that will cause the increase in dimension of the state space. Theoretical issues of adaptive control in the dynamic environment with time-varying structure of state spaces of plant and environment are not fully understood since the relevance of realizations was limited by technical capacities.

In [1], it is shown that traditional approaches to the model description are inefficient for nonequilibrium processes since the structure of the state space may change with time. There are experimental observations illustrating the presence of changing subsystems at mesoscopic scale (between the micro and macro) in nonequilibrium processes. Examples include the clustering in the flow of concentrated disperse mixtures, the formation of multi-scale vortex structures in turbulent liquid flows and plastic flow of solids under impulsive loading, as well as the hierarchy of structures in living systems. In case of open thermodynamic systems the synergistic processes of self-organization at the mesoscale level (due to the internal control feedback) together with an external disturbance (or control) lead to the emergence of new dynamic structures which are, in fact, discretizations of the space and time of nonequilibrium systems.

In [2], the networked systems of control and distributed parameter systems are considered as instances of dynamical systems, distributed along the discrete and continuum space, respectively. This unified perspective provides insightful connections, and gives rise to new questions in both areas. Among new directions in the research of the distributed systems the deep inner connections were outlined between the theory of the distributed systems and problems of turbulence and statistical mechanics.

The multiagent technology can effectively solve numerous problems arising in the distributed and nonstationary systems by replacing the general model of the interactions in a complex system with a set of local models. The problems of distributed interaction in dynamical networks has attracted a lot of attention in the last decade (see surveys [3]–[5]). This interest has been driven by applications in various fields, including the information exchange in multiprocessor networks, transportation networks, production networks, sensor and wireless networks, coordinated motion for unmanned flying vehicles, submarines and mobile robots, distributed control systems for power networks, complex crystal lattices, and nanostructured plants.

For the new applications of the theory to the flight control based on a vast array of the sensors and actuators distributed over the aircraft surface we use Local Voting (LV) control protocol with nonvanishing step-size, justified for the conditions of significant uncertainty and external disturbances [6]. This stochastic gradient-like (stochastic approximation) method has also been used before in other works (see, e.g. [7], [8]) but with the decreasing to zero step-size. Usually, the stochastic approximation is studied for the unconstrained optimization problems, but the above-mentioned results stimulated the development of new approaches [9] to track the changes in the parameter drift using the simultaneous perturbation stochastic approximation [10].

The paper is organized as follows. In Section II we describe the nonequilibrium statistical mechanics approach for the airplane flying in the turbulence flow. The model of an airplane with "feathers" is considered in Section III.

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In Section IV the Local Voting Protocol is presented with main assumptions and main results. Section V contains the concluding remarks.

II. Nonequilibrium Statistical Mechanics Approach for the Airplane in a Turbulence Flow

In the real world, any system consists of a huge amount of elementary units with complicated interconnections. The traditional methods for defining dynamical systems are based on a state space $\mathbb{X} \subset \mathbb{R}^{dim}$, $dim \in \mathbb{N}$, and dynamic equations

$$\dot{x} = A(x, u, w, \theta),\ x(t) \in \mathbb{X} \tag{1}$$

describing the time-evolution of the state vector $x(t) \in \mathbb{X}$. Hereinafter the symbol dot is used for the operation of differentiation over time. Control vector u is a controllable external input which, in simple words, plays a role of "an intermediary" between the system and the environment, while the uncontrollable disturbances are represented by the vector w. The form of the dynamical equations is usually defined by a certain finite set of the system parameters $\theta \in \Theta$. In general, the state vector $x(t)$ represents some integral characteristics related to the whole groups of the system elementary units. For example, position, velocity, and rotation of groups of particles moving together can be approximately described as an ideal solid body, or a set of similar characteristics for a group of bodies and properties of the interconnections links. The units with the similar behavior are usually called *"Cluster"*. In the laminar wind flow an aircraft is considered as an ideal solid moving in the flow with integral characteristics as a one cluster. Its dynamics can be considered in $\mathbb{R}^{10}$ (position, velocity, orientation and engine thrust associated with the center of gravity). The pressures on its wings surface can be considered as nominal. If we consider the wings surface divided by elementary units ("feathers") with the similar orientations (see Fig. 1), then all feathers refer to the same cluster, they have similar values (zero) of wind pressure deviations from the nominal values (it corresponds to the green color).

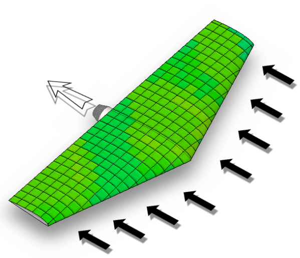

Fig. 1: The aircraft in a laminar wind flow.

The airplane motion in an ideal gas at velocity $\mathbf{V}(t)$ is governed by the equation

$$M\frac{d\mathbf{V}(t)}{dt} = \mathbf{F}_e + \mathbf{F}_d + \mathbf{F}_g + \mathbf{F}_l, \tag{2}$$

where M is the total airplane's mass, $\mathbf{F}_e$, $\mathbf{F}_d$, $\mathbf{F}_g$, $\mathbf{F}_l$ are the forces acting on the body: engine thrust, drag, gravity and lift, respectively.

If the velocity $\mathbf{V}(t)$ is constant: $\mathbf{V}(t) = \mathbf{V}_0$, and the airplane moves along the first axis $\mathbf{V}_0 = (V_0, 0, 0)$, then $\frac{d\mathbf{V}(t)}{dt} = 0$ all forces are counterbalanced.

The flow field around the airplane of a given shape is governed by the transport equations for the mass density and mass velocity ρ_0, v_0

$$\begin{aligned}(\mathbf{v}_0 \cdot \nabla)\rho_0 + \rho_0 \nabla \cdot \mathbf{v}_0 &= 0, & (3)\\ \rho_0(\mathbf{v}_0 \cdot \nabla)\mathbf{v}_0 &= -\nabla p_0 & (4)\end{aligned}$$

under the impermeability boundary conditions. For a body of the given shape all hydrodynamic fields are assumed to be known.

Each system communicates with the environment e. Sometimes environment $e(t)$, that is influencing the system, may be considered as a part of uncontrollable disturbances $w(t)$ (see Eq. (1)). Serious difficulties arise when at given moments of time $T_0, T_1, \ldots, T_m, \ldots$ the structure $s_0, s_1, \ldots, s_m, \ldots$ of environment disturbances $e(t)$ changes significantly. We make the assumption that the structure changes occur sufficiently infrequent, i.e. $\zeta = \min_m |T_{m+1} - T_m|$ is sufficiently large. Fig. 2 shows the different pressure deviations (different colors) for different units of airplane in case of a turbulent wind flow when all feathers remain the same (equal) orientations.

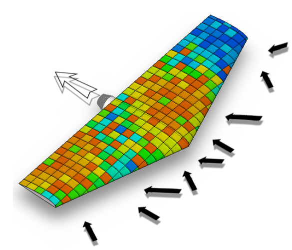

Fig. 2: The aircraft in a turbulence wind flow.

If the hydrodynamic field is slightly perturbed by the pressure field p_m in turbulent wind flow, the linearized equations for the small perturbations $\rho_m(\mathbf{r}, t), \mathbf{v}_m(\mathbf{r}, t)$ are nonstationary

$$\begin{aligned}\frac{\partial \rho_m}{\partial t} + (\mathbf{v}_0 \cdot \nabla)\rho_m + \rho_0 \nabla \cdot \mathbf{v}_m &= 0, & (5)\\ \rho_0 \frac{\partial \mathbf{v}_m}{\partial t} + \rho_0(\mathbf{v}_0 \cdot \nabla)\mathbf{v}_m &= -\nabla p_m. & (6)\end{aligned}$$

The nonstationary flow affects the body movement

$$M\frac{d\mathbf{V}_m(t)}{dt} = \mathbf{F}_e^m + \mathbf{F}_d^m + \mathbf{F}_g + \mathbf{F}_l^m \neq 0. \tag{7}$$

The set of linear equations (5)–(6) adequately describes the flow fields perturbations only in case of small spatial gradients and low speed of their changes, i.e. near the state of local equilibrium.

Starting from the Renaissance and up to the second part of the XX century the linear approach was dominant in the engineering. This approach is based on the assumption that the net effect is equal to the sum of all individual effects, and that the response to the effect is directly proportional to the effect. For the linear systems the effect is uniquely determined by the cause. So, linear mathematical models imply unambiguous determinism. In the framework of such models only the stable processes are studied. The stability problem is considered with a small disturbances in the linear laws. Such processes occur in systems near the thermodynamic equilibrium and are well reproduced in experiments

However, in [1] it is shown that the above set-up is often too stiff to accommodate a dynamic "transitional" processes in the systems caused by the rapid change of external conditions. The high-rate processes can decline the system state far from the equilibrium. Under the nonequilibrium conditions the linear approach fails and the effect starts to deviate from the cause and can be spread over space. The behavior of even simply constructed nonequilibrium systems can be very complex. The dynamic complexity can arise due to the multiple interactions of open system with its surrounding.

If the perturbed flow field changes rapidly in both space and time, the usual hydrodynamic equations (5)–(6) are no longer valid for calculations of the additional nonequilibrium forces (7) affecting the body. The nonequilibrium statistical mechanics states that the general transport equations are not entirely localized under essentially nonequilibrium conditions [11], [12]. Due to the incomplete description of high-rate processes, the most fundamental result is given by the general integral relationships between the conjugate thermodynamic fluxes and forces (gradients of macroscopic variables), which remains valid far from equilibrium throughout the relaxation regimes. Within the conventional continuum mechanics the momentum flux Π is a stress tensor, which consists of two parts $\Pi = -p\mathrm{I} + \mathrm{P}$ where p is the pressure, I is the unit tensor (reversible part), and P is the viscous stress tensor (irreversible part). Far from equilibrium the stress tensor cannot be separated into reversible and irreversible parts. In case of the rapid perturbations the pressure changes are more important than the slow momentum diffusion related to viscous effects. Then the integral expression for the momentum flux in terms of the pressure perturbations takes a form

$$\Pi(\mathbf{r},t) = -\int_0^t dt' \int_{Vol} d\mathbf{r}' \Re(\mathbf{r},\mathbf{r}',t,t') \bar{p}_m(\mathbf{r}',t')\mathrm{I}. \quad (8)$$

The integral kernel $\Re(\mathbf{r},\mathbf{r}',t,t')$ is the space-time correlation function averaging the large pressure gradients of the macroscopic fields and forming cluster's structures at the mesoscale level. With the substitution of $\bar{p}_m$ instead of p_m, equations (5)–(6) become integro-differential and, in contrast to the differential continuous mechanics models, are valid under nonequilibrium conditions. These updated equations take the memory and space non-local effects into account. Being constructed in the framework of the nonlocal transport theory [13], [14], the model expression of the kernel responsible for the structure formation under the impulse external action

$$\Re_s(\mathbf{r},\mathbf{r}',t) = \frac{1}{q(\mathbf{r},t)} \exp\left\{-\frac{\pi\,(\mathbf{r}'-\mathbf{r})^2}{q^2(\mathbf{r},t)}\right\} \quad (9)$$

involves the parameter q, which represents a linear size of the structure element. In case of the structureless medium, the averaging scale tends to zero: $q \to 0$, the kernel function tends to the Dirac's $delta$-function, and the governing equations become differential and correspond to (5)–(6).

III. An Airplane with "Feathers"

The above studies shows that we can manipulate the system elements at the mesoscale level in order to reduce the forces affecting the system due to the external perturbations.

For the airplane flying in a turbulent flow, let's assume that we can use a sort of "feather"-elements which are able to change the local fluxes around the plane surface and, in particular, equalize deviations of the pressure on the upper surface of the airplane. In this case the same reaction of feathers in the same cluster to the external disturbances should be expected. The macroscopic reaction of a medium to the external disturbances should be entirely determined by the time evolution of the finite-size correlations. Moreover, the system could demonstrate effects of its internal structure such as clustering, presented in Fig. 3.

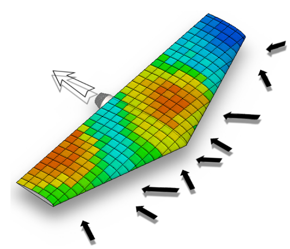

Fig. 3: The aircraft in a turbulence wind flow. Clustering of "feathers".

Once the pressure deviation $p_m(\mathbf{r}',t)$, generated by the external turbulent flow, significantly changes at the time instants $T_0, T_1, \ldots, T_m, \ldots$, by virtue of (8), we get

$$\tilde{p}(\mathbf{r},t) = \int_{Vol} d\mathbf{r}' \Re_s(\mathbf{r},\mathbf{r}',t) p_m(\mathbf{r}',T_m) \quad (10)$$

where $t \in (T_m, T_{m+1})$, $\tilde{p}(\mathbf{r},t)$ is the partially equalized pressure deviation due to the "feathers" collective action. If the value ζ is sufficiently large then the equalization process is completed during a time interval $\tau << \zeta$ when $\tilde{p}(\mathbf{r},t) \to \bar{p}^u_m = const$ for "feathers" on the upper surface and $\tilde{p}(\mathbf{r},t) \to \bar{p}^l_m = const$ for "feathers" on the lower surface.

As far as the integral kernel $\Re$ in (10) is determined by the averaging process, for the discrete feathers the integration is replaced by the sum averaging the pressure deviations p^j_t over the cluster i at time instant t

$$\bar{p}^i_t = \frac{1}{|N^i_t|} \sum_{j \in N^i_t} p^j_t = p^i_t + \frac{1}{|N^i_t|} \sum_{j \in N^i_t} (p^j_t - p^i_t) \quad (11)$$

where N_t^i is a set of indexes which belong the same cluster i, $|N_t^i|$ is a number of elements in the cluster i. Hereandfuther the upper index i denotes the number of feather.

In order to answer the question what internal control law governs the pressure deviation equalization, let us calculate the full energy of the perturbations. For subsonic speed the full kinetic energy of turbulent perturbations at the time instant t is a sum of energies generated by the mass velocity perturbation field near each feather

$$E = \sum_i E^i = \sum_i \frac{1}{2}\rho_0^i (v_m^i)^2.$$

According to the Bernoulli's theorem the pressure perturbation near each feather can be evaluated in the linear approximation as follows $p_m^i = \rho_0^i v_0^i v_m^i$. Then, the sum energy of turbulent perturbations at the time instant t can be expressed through the pressure perturbations near each feather

$$E = \sum_i E^i = \sum_i \frac{1}{2\rho_0^i (v_0^i)^2}(p_m^i)^2.$$

During the pressure deviation equalization the pressure perturbation near i-th feather is replaced by its averaged value $\bar{p}_t^i$ according to (11). Then the discrete form of the energy perturbation for an element i is

$$E_t^i = \frac{\left[p_t^i + \frac{1}{|N_t^i|}\sum_{j\in N_t^i} p_t^j - p_t^i\right]^2}{2\rho_0^i (v_0^i)^2}. \tag{12}$$

The full energy of the perturbation fields is a sum

$$E_t = \sum_i E_t^i = \sum_i \frac{\left(\bar{p}_t^i\right)^2}{2\rho_0^i (v_0^i)^2}. \tag{13}$$

Minimization of the sum perturbation energy according to Speed Gradient algorithm in the finite form [15], [16] gives an equation governing the pressure equalization process

$$\dot{\bar{p}}_t^i = const - \gamma \frac{\partial \dot{E}_t}{\partial \dot{\bar{p}}_t^i} \tag{14}$$

with gain parameter γ and

$$\dot{E}_t = \frac{dE}{dt} = \sum_i \frac{1}{\rho_0^i (v_0^i)^2}\bar{p}_t^i \cdot \dot{\bar{p}}_t^i.$$

For the pressure difference $\Delta\bar{p}_t^i = \bar{p}_t^i - \bar{p} \xrightarrow[t\to\infty]{} 0$ Eq. (14) takes a form

$$\Delta\dot{\bar{p}}_t^i = -\gamma \frac{1}{\rho_0^i (v_0^i)^2}\Delta\bar{p}_t^i. \tag{15}$$

The solution of Eq. (15) is

$$\Delta\dot{\bar{p}}_t^i = \Delta\dot{\bar{p}}_{t=0}^i \exp\left\{-\gamma \frac{t}{\rho_0^i (v_0^i)^2}\right\} \xrightarrow[t\to\infty]{} 0.$$

It defines a typical equalization time $\tau = \max \rho_0^i (v_0^i)^2/\gamma$.

Note. We have got the expression for transition time τ. When the pressure equalization is rapid, and the equalization time is small compared to the time interval between the changes of the external turbulent flow $\tau << T_{m+1} - T_m$, then at the time interval $[T_m + \tau, T_{m+1}]$ the most part of the system units (feathers) has already clustered. The total number of clusters is much less than the total number of the system units. In particular, the only one cluster with the same pressure additives can be formed. We can define new state variables for each cluster and consider the new system model for such cluster (integral) variables. The dimension of this new model will be significantly less then the original one. This model will be valid during the time interval $[T_m + \tau, T_{m+1}]$, and it can be linearized for many practical cases. However, this simplified model will "operate" only for a finite time interval as long as there will be no further changes in the structure of external disturbances. The considered dynamic behavior has common features with many others complicated problems. The new possible general framework is discussed in [17] based on cluster flows.

In the next section we propose the possible way to realize the considered internal control law for an airplane with "feathers", and we derive the upper bound for the transition time in the proposed scheme.

IV. Local Voting Protocol

We consider the division of the upper airplane surface into n finite elements ("feathers") $a^1, \ldots, a^n$ (the similar scheme is also valid for the lower airplane surface). The plane of "feather" may change its angle (rotate) within some boundaries. Let us assume that during the short time interval δ the "feathers" are able to intercommunicate with their neighbors and slightly vary their angles. For each "feather" $i \in N = \{1, 2, \ldots, n\}$ and time instant $t_k = T_m + \delta k,\ k = 0, 1, \ldots, [(T_{m+1} - T_m)/\delta]$, we denote the average pressure deviation for the "feather"-element a^i as x_k^i and the set of its neighbors as N_k^i. The internal control law (14) takes a discrete form

$$x_{k+1}^i = x_k^i + \gamma \sum_{j\in N_k^i} b_k^{i,j}(y_k^j - y_k^i) \tag{16}$$

with some gain coefficient (step-size) γ, weight coefficients $b_k^{i,j}$, initial conditions

$$x_0^i = \int_{a^i} p_m(\mathbf{r}, T_m)d\mathbf{r}, \tag{17}$$

and observations

$$y_k^i = x_k^i + \xi_k^i,\ i \in N, \tag{18}$$

which contain the error values (noise) ξ_k^i. Equation (16) is called a *Local Voting Protocol* [6]. We set $b_k^{i,j} = 0$ for other pairs (i, j) when $j \notin N_k^i$. The matrix of the control protocol is denoted by $B_k = [b_k^{i,j}]$. The corresponding graph with adjacency matrix B_k is denoted by $\mathcal{G}_{B_k}$.

A. Consensus Problem

The behavior investigation problem of partially equalized pressure deviation x_k^i (due to the collective action of "feathers") can be reformulated as a *consensus* problem.

Definition *1:* The network is said to achieve a *consensus* at the time instant k if there exists a variable $\bar{x}$ such that $x_k^i = \bar{x}$ for all $i \in N$.

Though consensus is our desired goal, it cannot be practically achieved by virtue of the discretization and approximation assumptions alone. For the more realistic problem statement let us consider a *ε-consensus* achievement. Further layouts assume the statistical nature of uncertainties.

Let $(\Omega, \mathcal{F}, \mathbf{P})$ be the underlying probability space corresponding to the sample space, the collection of all events, and the probability measure respectively, E is a symbol of the mathematical expectation.

Definition *2:* The network is said to achieve *a probability ε-consensus with confidential level* $\beta \in [0,1]$ at time k if there exists a variable $\bar{x}$ such that $\mathbf{P}\{\|\mathbf{x}_k - \bar{x}\mathbf{1}\|^2 \le \varepsilon\} \le \beta$ for all $i \in N$ where $\mathbf{x}_k$ is the column of $x_k^1, x_k^2, \ldots, x_k^n$ and $\mathbf{1}$ is the column of ones.

Definition *3:* $K(\varepsilon)$ is called *a time to the probability ε-consensus with confidential level* $\beta \in [0,1]$, if the network achieves probability ε-consensus with confidential level $\beta \in [0,1]$ for all $k \ge K(\varepsilon)$.

To introduce some properties of the network topology, the following definitions from the graph theory will be used. *The weighted in-degree* of node i is defined as the sum of i-th row of matrix B: $d^i(B) = \sum_{j=1}^n b^{i,j}$; $\mathrm{diag}\{d^i(B)\}$ is the corresponding diagonal matrix; $d_{\max}(B)$ is the maximum in-degree of graph $\mathcal{G}_B$; $\mathcal{L}(B) = \mathrm{diag}\{d^i(B)\} - B$ is the *Laplacian* of graph $\mathcal{G}_B$; $\cdot^{\mathrm{T}}$ stands for a vector or matrix transpose operation; $\|B\|$ is the Euclidian norm: $\|B\| = \sqrt{\sum_i \sum_j (b^{i,j})^2}$; $Re(\lambda_2(B))$ is the real part of the second eigenvalue of matrix B ordered by absolute magnitude; $\lambda_{\max}(B)$ is the maximum eigenvalue of matrix B. Digraph $\mathcal{G}_{B_{sg}}$ is said to be a subgraph of the digraph $\mathcal{G}_B$ if $b_{sg}^{i,j} \le b^{i,j}$ for all $i, j \in N$. Digraph $\mathcal{G}_B$ is said to contain a *spanning tree* if there exists a directed tree $\mathcal{G}_{tr}$ as a subgraph of $\mathcal{G}_B$.

B. Assumptions and Theorems

We assume that the following conditions are satisfied.

A1: a) For all k and $i \in N$ observations noises ξ_k^i are zero-mean, independent identically distributed (i.i.d.) random variables with bounded variances: $\mathrm{E}(\xi_k^i)^2 \le \sigma_\xi^2$. **b)** For all $i \in N$ and $j \in N_{\max}^i = \cup_k N_k^i$ the appearance of "variables" edges (j,i) in the graph $\mathcal{G}_{B_k}$ is independent random event and weights $b_k^{i,j}$ in (16) are independent random variables with expectations: $\mathrm{E}b_k^{i,j} = \bar{b}^{i,j}$, and bounded variances: $\mathrm{E}(b_t^{i,j} - \bar{b}^{i,j})^2 \le \sigma_b^2$. Additionally, all these random variables are mutually independent.

A2: Graph $\mathcal{G}_{\bar{B}}$ has a spanning tree, $\bar{B} = \mathrm{E}B_k$.

A3: For the step-size $\gamma > 0$ the following conditions are satisfied: $\gamma \le \frac{1}{d_{\max}(\bar{B})}$, and $\psi(\gamma) = \gamma Re(\lambda_2(\bar{B})) - \gamma^2 \lambda_{\max}(Q) > 0$ where $Q = \mathrm{E}(\mathcal{L}(\bar{B}) - \mathcal{L}(B_k))^{\mathrm{T}}(\mathcal{L}(\bar{B}) - \mathcal{L}(B_k))$.

Note. If matrix B_k does not vary on k, then $B_k \equiv \bar{B}$, $\psi(\gamma) = \gamma Re(\lambda_2(\bar{B}))$, and condition $\psi(\gamma) > 0$ holds for all γ since $Q = 0$.

Theorem *1:* **If** Assumptions **A1–A3** are satisfied, $\beta \in (0,1)$, and

$$\varepsilon = \frac{\Delta(\gamma) + (1-\psi(\gamma))^{k-1}\left(\|\mathbf{x}_0 - \bar{x}\mathbf{1}\|^2 - \Delta(\gamma)\right)}{1-\beta}, \quad (19)$$

where $\Delta(\gamma) = 2\sigma_\xi^2\gamma^2(n^2\sigma_b^2 + \|\bar{B}\|^2)/\psi(\gamma)$, $(\Delta(\gamma) = 2\sigma_\xi^2\gamma(n^2\sigma_b^2 + \|\bar{B}\|^2)/Re(\lambda_2(\bar{B}))$ if $B_k \equiv \bar{B})$, **then** system (16)–(18) achieves the probability ε-consensus with confidential level β for weighted average of the initial states

$$\bar{x} = \frac{\sum_i g^i x_0^i}{\sum_i g^i} \quad (20)$$

where $(g^1, g^2, \ldots, g^n)^T$ is the left eigenvector of matrix $\bar{B}$ [5] ($\bar{x} = \frac{1}{n}\sum_{i=1}^n x_0^i$ in the case of balanced topology graph $\mathcal{G}_{\bar{B}}$).

Proof: All conditions of Theorem 1 from [18] hold under appropriate notations. Hence, we have

$$\mathrm{E}\|\mathbf{x}_k - \bar{x}\mathbf{1}\|^2 \le \Delta(\gamma) + (1-\psi(\gamma))^{k-1}\left(\|\mathbf{x}_0 - \bar{x}\mathbf{1}\|^2 - \Delta(\gamma)\right).$$

Using this inequality and applying the Markov's inequality, we derive

$$\mathbf{P}\{\|\mathbf{x}_k - \bar{x}\mathbf{1}\|^2 > \varepsilon\} \le 1 - \beta$$

i.e. the probability ε-consensus with confidential level β is achieved. ■

Eq. (19) gives a way to choice of an optimal step-size for consensus protocol. In changing conditions it is not possible to get it but it may be useful to track it via stochastic approximation type algorithm as in [19].

Theorem 1 gives lower bound $\underline{\varepsilon} = \frac{\Delta(\gamma)}{1-\beta}$ for the possible ε level of the probability ε-consensus with confidential level β. Now we estimate the time to the probability ε-consensus with confidential β for $\varepsilon > \underline{\varepsilon}$.

Theorem *2:* **If** all conditions of Theorem 1 hold,

$$\tilde{x}_0 = \begin{cases} \ln(\|\mathbf{x}_0 - \bar{x}\mathbf{1}\|^2 - \Delta(\gamma)), & \text{if } \|\mathbf{x}_0 - \bar{x}\mathbf{1}\|^2 > \Delta(\gamma), \\ 0, & \text{otherwise,} \end{cases}$$

and $\varepsilon > \underline{\varepsilon}$, **then** inequity

$$K(\varepsilon) \ge \frac{\ln(\varepsilon - \underline{\varepsilon}) - \tilde{x}_0}{\ln(1-\psi(\gamma))}, \quad (21)$$

holds for the time to the probability ε-consensus with confidential level β.

Proof: The result of Theorem 2 follows immediately from the expression (19). ■

Inequality (21) allows calculation of the transition time τ for equalizing the pressure after the changing of structure of the wind flow. We have

$$\tau \approx \delta K(\varepsilon)$$

with predefined confidential parameter $\beta \in (0,1)$ and approximately consensus level ε. Hence, for the practical applicability of described scheme we need to check that

$$\delta K(\varepsilon) << \zeta.$$

Note. The important feature of the control protocol (16) is that there is no any special tools of "fault analysis" as, for example, in [20] to determine the time instant T_m, at which the structure of the incoming flow changes. Feathers begin to automatically adjust according to the feedback in violation of consistency pressures deviations from the nominal values.

C. Stand for Simulation

We developed a stand based on the frame of flying wing drone to test the algorithm. The frame had 1.5 m wingspan, 0.5 m length and from 10 to 35 mm in the thickness. Almost all of the drone surface was covered by plates ("Feathers") with servos. The number of plates used was 100 pcs. The size of the single plate was 60 mm $\times$ 60 mm in maximum. The plates were uniformly placed over the surface. Depending on the location on the drone surface the plates had different sizes and forms. The plates were moved by servos (Fig. 4) to change the air flow pressure on them. We used the square force-sensitive resistor to measure the pressure. Data acquisition and generation of the PWM control signal were carried out by the development board Arduino Mega 2560 R3. One Arduino 2560 provided the control of 14 servos and was able to collect the data from 16 analog sensors. For our stand we used 7 boards in total. All information from Arduino was collected in the main computer where we organized the interconnection between the nearest neighbors' plates and the algorithm of interaction. At current stage of our project it allows us to get performance analysis for different kinds of network topology. Six flexible powerful fans were used to create the air flow in the test room. In addition, we used the LEDs, which were installed on the plates, for the indication of consensus achievement status. In the future, we plan to design a microcomputer to be installed on every plate and to implement the interconnection without the main computer and Arduino boards.

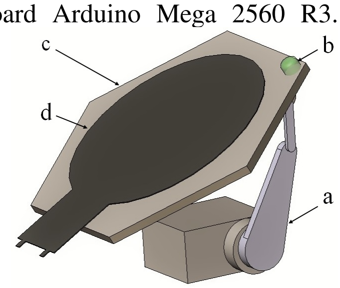

Fig. 4: Elements of "feather". a – servo, b – light-emitting diode (LED), c – base, d – force-sensitive resistor.

V. Conclusions and Future Work

Miniaturization of control plants and high frequency control actions do not allow a verification of the model of the movement with the traditional high degree of accuracy. This fact emphasizes the key problem of adaptive control development in presence of significant uncertainties and external disturbances. As noted in the end of Section III, this problem is more complicated in the context considered throughout this paper since we have only finite time interval for the adaptation process. In case of the state space structure changing with time, we plan to extend our results for the parameter tracking using SPSA (Simultaneous Perturbation Stochastic Approximation) [9] and LSCR approach (Leave-out Sign-dominant Correlation Regions) [21], which was proposed earlier by M. Campi with co-authors to increase the effectiveness of adaptive control based on finite (not large) set of experimental data only [22].

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